

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequations [2022]

Level - 2

Daily Tutorial Sheet 11

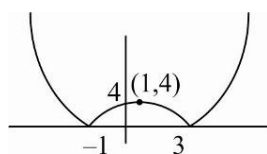
Paragraph for Q.126 to 128

126.(C)

127.(B)

128.(C)

$$x^2 - 2x - 3 = 0 \Rightarrow x = 3, -1$$

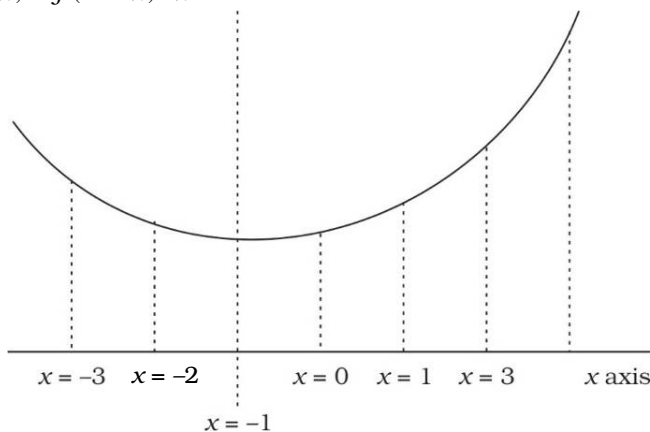


graph of $y = |x^2 - 2x - 3|$

For four solutions $m \in (0, 4)$, For three solutions $m = 4$ and For two solutions $m \in (4, \infty) \cup \{0\}$

Paragraph for questions Nos. 129 to 130

129.(C) $f(-1-x) = f(-1+x) \forall x \in \mathbb{R}$



$\Rightarrow$ Graph of $f(x)$ is symmetric about $x = -1$.

$$-\frac{b}{2a} = -1 \Rightarrow b = 2a$$

$$\alpha = f(-2) = 4a - 2b + c ; \beta = f(3) = 9a + 3b + c ; \gamma = f(-3) = 9a - 3b + c$$

Using graph $f(3) > f(-3) > f(-2) \Rightarrow \beta > \gamma > \alpha$

130.(A) $p = b - 4a = -2a < 0$

$$q = 2a + b = 4a > 0 \Rightarrow p \times q < 0$$

Paragraph for questions Nos. 131 to 132

131.(A), 132.(C)

$$f(x) = (k+1)x^2 - (20k+14)x + (91k+40) = 0$$

$$\Rightarrow k(x^2 - 20x + 91) + (x^2 - 14x + 40) = 0$$

$\Rightarrow k(x-7)(x-13) + (x-4)(x-10) = 0$
 Let $g(x) = k(x-7)(x-13)$ and $h(x) = (x-4)(x-10)$
 $g(x) < 0$ iff $x \in (7, 13)$ and $h(x) < 0$ iff $x \in (4, 10)$
 Clearly, $f(4) > 0$, $f(7) < 0$ and $f(10) < 0$, $f(13) > 0$
 Hence, $\alpha \in (4, 7)$ and $\beta \in (10, 13)$

Paragraph for questions Nos. 133 to 135

133.(C) $f(x) = x^2 + bx + c \forall x \in R$

Least value at $\frac{-b}{2} = -1 \Rightarrow b = 2$

Graph of $f(x)$ cuts y -axis, when $x = 0$

$$\Rightarrow c = 2 \Rightarrow f(x) = x^2 + 2x + 2$$

Least value of $f(x) = 1$

134.(D) $f(-2) + f(0) + f(1) = 9$

135.(A) $a \in (1, \infty)$

Paragraph for questions Nos. 136 to 137

136.(D) $(\log_2 x)^2 - 4(\log_2 x) - m^2 - 2m - 13 = 0$

$$D > 0 \Rightarrow m^2 + 2m + 17 > 0 \forall m \in R$$

137.(D) $m^2 + 2m - (\log_2 x)^2 + 4(\log_2 x) + 13 = 0$

$$D \geq 0 \Rightarrow (\log_2 x - 6)(\log_2 x + 2) \geq 0 \Rightarrow x \in \left(0, \frac{1}{4}\right] \cup [64, \infty)$$

$$\text{Alternate solution: } \log_2 x = \frac{4 \pm \sqrt{16 + 4(m^2 + 2m + 13)}}{2}$$

$$\Rightarrow \log_2 x_1 = 2 - \sqrt{m^2 + 2m + 17} \text{ and } \log_2 x_2 = 2 + \sqrt{m^2 + 2m + 17}$$

$$\text{Now } m^2 + 2m + 17 = (m+1)^2 + 16 \in [16, \infty)$$

$$\Rightarrow \log_2 x_1 \in (-\infty, -2] \text{ and } \log_2 x_2 \in [6, \infty) \Rightarrow x_1 \in (0, \frac{1}{4}] \text{ and } x_2 \in [64, \infty)$$

Paragraph for questions Nos. 138 to 139

138.(B), 139.(D)

$x^4 - 2x^3 - 3x^2 + 4x - 1 = 0$ has four roots $a, \frac{1}{a}, b, \frac{-1}{b}$ (because product of all 4 roots = -1 and product of 1 pair = 1)

$$\left(a + \frac{1}{a}\right) + \left(b - \frac{1}{b}\right) = 2 \quad \dots(1)$$

$$\left(b - \frac{1}{b}\right) - \left(a + \frac{1}{a}\right) = -4 \quad \dots(2)$$

{Taking sum of multiples of roots taken 3 at a time}

$$\Rightarrow a + \frac{1}{a} = 3 \text{ and } b - \frac{1}{b} = -1 \Rightarrow a^2 - 3a + 1 = 0 \text{ and } b^2 + b - 1 = 0 \Rightarrow a = \frac{3 \pm \sqrt{5}}{2} \text{ and } b = -\frac{1 \pm \sqrt{5}}{2}$$

Hence, we have obtained all 4 roots and since $x_1 < x_2 < x_3 < x_4$,

$$x_1 = \frac{-1 - \sqrt{5}}{2} = \frac{-1}{b}, x_2 = \frac{3 - \sqrt{5}}{2} = \frac{1}{a}, x_3 = \frac{\sqrt{5} - 1}{2} = b, x_4 = \frac{3 + \sqrt{5}}{2} = a.$$

$$x_2 x_4 = 1 \quad x_1 x_3 = -1 \quad x_1 x_2 = \frac{-1}{ab} \quad x_3 x_4 = ab.$$

$$x_2 x_4 + x_1 x_3 + x_1 x_2 + x_3 x_4 = ab - \frac{1}{ab}$$

$$ab = \frac{\sqrt{5} + 1}{2} \Rightarrow \frac{1}{ab} = \frac{\sqrt{5} - 1}{2} \cdot ab \cdot \frac{-1}{ab} = 1$$

$$x_2^3 + x_4^3 = a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right) \left[a^2 - \frac{1}{a} = -1\right] = \left(a + \frac{1}{a}\right) \left[\left(a + \frac{1}{a}\right)^2 - 3\right]$$

$$a + \frac{1}{a} = 3 \Rightarrow a^3 + \frac{1}{a^3} = 18$$